Matrix computations with applications

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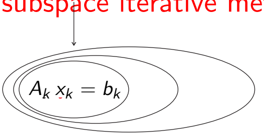
Problems of interest

- Linear algebraic systems $Ax = b$, $A \in \mathbb{R}^{n\times n}$ nonsingular:
  discretization of differential or integral equations from modelling, e.g., in material science, continuum mechanics
- Linear approximation problems $Ax \approx B, A \in \mathbb{R}^{n\times m}, B \in \mathbb{R}^{n\times d}$:
  errors-in-variables modeling, e.g, in statistical applications
- Linear ill-posed problem $Ax \approx b$:
  image processing (medical, radar, sonar imaging, astronomical observations), bioelectrical inversion problems, seismology

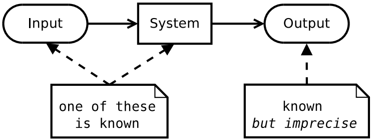
Often, A (representing a model) is large and sparse, thus Krylov subspace iterative methods are used to solve them,



$$\mathcal{K}_k(A, b) = Span\{b, Ab, \ldots, A^{k-1}b\}.$$

Inverse problems in image processing

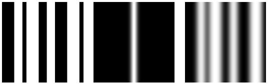
Inverse Problem



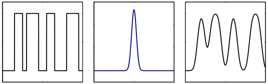
Inverse problems are often modeled by a Fredholm integral of the first kind with a kernel $K(s,t)$ having a smoothing property,

$$g(s) = \int_I K(s,t) f(t) \mathrm{d}t.$$

Example: barcode reading



sharp kernel blurred



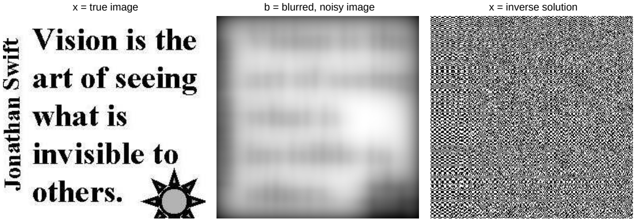
$f(t)$ $K(s,t)$ $g(s)$

Naive solution

Discretization leads to complicated noise contaminated problems

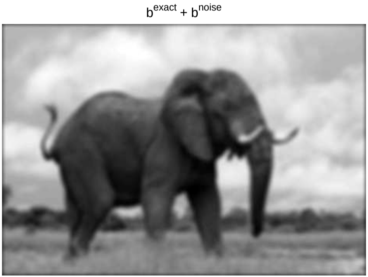
$$b \approx Ax + e.$$

The presence of noise in the measured data and the properties of the problem result in “naive” solution $x := A^{\dagger}b$ that is meaningless.

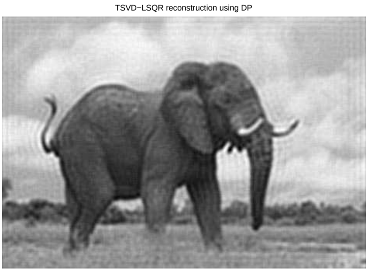


Regularization and denoising

Theoretical analysis of Krylov subspace method – Golub-Kahan iterative bidiagonalization led to cheap estimator of the unknown noise. This allows to denoise the problem.



$n = 152280$, 10% noise



reconstruction

Research projects

- Image Processing in Jewellery Industry with Emphasize on Defect Analysis: financed by Preciosa a.s. (2014 - 15)
  $\rightarrow$ image processing methods in detection of defects in the produced stones

- Iterative Methods in Numerical Mathematics: Analysis, Preconditioning, and Applications: GAČR grant (2013 - 17)
  $\rightarrow$ fundamental research of Krylov subspace methods

- University Center for Mathematical Modeling, Applied Analysis and Computational Mathematics: University Center of Excellence (2012 - 17)
  $\rightarrow$ apply state-of-art mathematical tools in applied sciences

- Necas Center for Mathematical Modeling: (2013 - ?)
  $\rightarrow$ theoretical and applied math in continuum mechanics

Thank you for your kind attention.