A light at the end of the tunnel

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Graded properties, expressed by graded predicates, are ubiquitous in human discourse and reasoning. They are characterized by the fact that utterances of (simple) sentences involving them are graded, that is, admit mutual comparison. Typical examples are vague properties (such as *red*, *old*, *tall*, *rich*), i.e. those that do not establish a clear distinction between objects that satisfy them and those that do not, and hence have borderline cases.[1] Mathematically, we may say that graded properties are those that can be modelled by a function from the universe of discourse to a set (or a set of tuples) of ordered grades. In the example, it could naturally be the mapping that sends each angle to its number of degrees.

Arguably, graded properties are epistemologically necessary, as they allow putting together many similar notions that would otherwise collapse the conceptual system (and the language) with too many properties (and predicates). It is an economic necessity to reason with one predicate *red*, instead of having infinitely many for each possible level in the color spectrum. Reasoning with graded properties is successfully and correctly carried out in many contexts (not withstanding the fact that natural language has enough devices to provide higher levels of precision whenever necessary).

Logic is the science of correct reasoning and, as such, has to deal with all forms of valid consequence potentially carried out by rational beings. During most of its history as a formal science, Logic has tried to explain correct reasoning by means of a classical paradigm, that of logic based on bivalence principle. Despite of its many merits and achievements, such approach does violence to many properties, forcing crisp definitions (splitting graded properties into many crisp ones) in contexts that did not require them. Indeed, natural language allows satisfactory communication and correct reasoning in terms of graded predicates. The classical logical analysis seems, therefore, too artificial and detached from actual reasoning.

However, to the best of our knowledge, it appears that a serious research program aimed at understanding reasoning with graded predicates in all its complexity from the point of view of Logic has never been carried out. Moreover, it has been argued by Stenning and van Lambalgen [2] that Logic, despite its origins, has become increasingly divorced from the

[1] But not only vague properties are graded. Take, for example, the notion of *acute angle*. It is clearly not vague, it has no borderline cases and its extension is crisply defined as the set of angles of strictly less than 90 degrees; however, if α and β are, respectively, angles of 30 and 89 degrees, then α is more acute than β which somehow makes the utterance 'α is acute' stronger than 'β is acute'.

study of correct reasoning and has become instead an application-oriented discipline mainly connected to Computer Science, while being replaced by Psychology and Cognitive Science in the study of reasoning. We share their conviction that Logic can (and should) recover its original motivations without losing its application potential. It is our belief that one reinforces the other. If Logic manages to give a satisfactory account of correct reasoning in all possible scenarios, then it will be more likely to provide useful applications to Computer Science by automatizing suitable fragments of correct reasoning.

We think that, rather than providing logical foundations to Fuzzy Set Theory, the real goal has in fact been all along (though maybe not explicitly said) the study of correct reasoning with graded predicates. On one hand, this is confirmed by its actual practice, the focus on logical systems in which the predicates are given a semantics over linearly ordered sets of degrees. On the other hand, as argued by Smith [1], MFL provides a good logical apparatus to explain reasoning with vagueness, with several advantages w.r.t. alternative approaches proposed in the philosophical discussions around theories of vagueness.

Moreover, if we are concerned with all graded predicates (not only vague predicates), we should leave behind the restriction to linearly ordered sets of grades. The only essential feature of grades must be that they are partially ordered. Whether the order is total or not will depend on the specific nature of each predicate under each semantical interpretation. In this way, one can see that our research enterprise encompasses most many-valued logics, not only the so-called fuzzy logics (or, more technically, *semilinear logics*). Also, since Algebraic Logic provides a method to endow any logical system with an algebraic semantics, most non-classical logics, insofar their algebras come with a partial order, will also fall under the scope of this program, for they may deal with graded predicates.

Therefore, we propose a very ambitious research program that should unfold in the following three layers:

1. Natural language and natural reasoning scenarios: Interdisciplinary research relating Logic to Cognitive Science, Psychology, Linguistics, and Philosophy, in order to understand how correct reasoning is carried out in natural language with graded properties. In particular, one should study the syntactical transformation of natural language arguments into a formal language amenable to by analyzed by logical tools.

2. Formal interpreted languages and artificial reasoning scenarios: Logical systems with specific interpretations for their formal syntax, coping with graded properties in specific contexts. This has a potential for applications to Computer Science.

3. Formal abstract languages and Mathematical Logic: Continuation of the mathematical study of general logical systems with a graded semantics.

References

[1] Nicholas J.J. Smith, *Vagueness and Degrees of Truth*, Oxford University Press, 2013.

[2] Keith Stenning and Michiel van Lambalgen *Human Reasoning and Cognitive Science*, The MIT Press, 2008.