Future of MFL: Fuzzy Natural Logic and AST
Abstract

Vilém NOVÁK and Irina PERFILIEVA

University of Ostrava
Institute for Research and Applications of Fuzzy Modeling
NSC IT4Innovations, 30. dubna 22, 701 03 Ostrava 1
Czech Republic
E-mail: Vilem.Novak@osu.cz

The research in MFL has been mainly focused on its metamathematics. But as proclaimed many times in introductions of almost all papers, MFL should serve as a logic providing tools for development of mathematical model of vagueness and tools for various kinds of applications that have to cope with the latter. It seems that this goal is not sufficiently carried out. Our suggestions is to focus more inside fuzzy logic and not on its outer properties. Below we outline one possible way.

Natural Logic and Fuzzy Natural Logic. In [4], Lakoff introduced the concept of *natural logic* that is defined as a collection of terms and rules that come with natural language and that allow us to reason and argue in it. *Fuzzy Natural Logic* (FNL) follows these ideas but the goal is to develop it as a *mathematical theory* fulfilling the following:

(a) Follows the goals of natural logic.

(b) Includes model of the vagueness phenomenon.

(c) Follows results of the logical analysis of natural language (see, e.g., [1]).

Till now, FNL consists of the (1) *formal theory of evaluative linguistic expressions* [12], (2) *formal theory of fuzzy/linguistic IF-THEN* rules, linguistic descriptions and approximate reasoning based on them [10, 16, 17] and (3) *formal theory of intermediate and fuzzy generalized quantifiers* [2, 5, 6, 13]. The formal tool for FNL is Łukasiewicz fuzzy type theory (a higher-order fuzzy logic; see [9, 14]).

Evaluative linguistic expressions are expressions of natural language, for example, *small, medium, big, roughly one hundred, very short, more or less deep, not tall, roughly warm or medium hot, quite roughly strong, roughly medium size*, etc. They form a small, syntactically simple, but very important part of natural language which is present in its everyday use any time. In FNL, a special formal theory of FTT has been constructed using which semantics of the evaluative expressions including their vagueness is modeled.

Fuzzy/linguistic IF-THEN rules are in FNL taken as genuine conditional clauses of natural language with the general form

$$\mathsf{IF}\ X \text{ is } \mathcal{A}\ \mathsf{THEN}\ Y \text{ is } \mathcal{B}, \tag{1}$$

where "X is $\mathcal{A}$", "Y is $\mathcal{B}$" are evaluative linguistic predications. A finite set of rules (1) is called *linguistic description* and it is construed as a special text. The method of *perception-based logical deduction* enables to derive conclusion from linguistic descriptions, thus simulating reasoning of people.

Intermediate and fuzzy quantifiers are natural language expressions such as *most, a lot of, many, a few, a great deal of, large part of, small part of.* In correspondence with [18] they are in FNL modeled as special formulas of fuzzy type theory in a certain extension of the formal theory of evaluative linguistic expressions. Typical elaborated quantifiers are

"Most (Almost all, Few, Many) B are A".

These quantifiers occur also in generalized Aristotle syllogisms where formal validity of more than 120 such syllogisms was proved.

Applications of FNL include models of commonsense human reasoning, managerial decision-making, linguistic control of processes, forecasting and mining information from time series and other ones.

MFL and AST? An exciting possibility is to switch the development of MFL to a new mathematical frame based on the Alternative Set Theory by P. Vopěnka [19]. Recall that this theory is based (among others) on different understanding to infinity. Vopěnka proclaims that mathematics uses infinity whenever it faces vagueness. His concept of semiset is essentially motivated by the latter. Old attempts to reconcile fuzzy sets with AST are contained in [7, 8].

References

[1] M. Duží, B. Jespersen, P. Materna: *Procedural Semantics for Hyperintensional Logic*, Springer, Dordrecht, 2010.

[2] A. Dvořák, M. Holčapek: *L-fuzzy quantifiers of the type* $\langle 1 \rangle$ *determined by measures*, Fuzzy Sets and Systems **160** (2009) 3425–3452.

[3] P. Hájek: *What is mathematical fuzzy logic*, Fuzzy Sets and Systems **157** (2006) 597–603.

[4] G. Lakoff: *Linguistics and natural logic*, Synthese **22** (1970) 151–271.

[5] P. Murinová, V. Novák: *A formal theory of generalized intermediate syllogisms*, Fuzzy Sets and Systems **186** (2012) 47–80.

[6] P. Murinová, V. Novák: *The structure of generalized intermediate syllogisms*, Fuzzy Sets and Systems **247** (2014) 18–37.

[7] V. Novák: *Fuzzy sets — the approximation of semisets.* Fuzzy Sets and Systems **14** (1984) 259–272.

[8] V. Novák: *The Alternative Mathematical Model of Linguistic Semantics and Pragmatics.* Plenum, New York, 1992.

[9] V. Novák: *On fuzzy type theory*, Fuzzy Sets and Systems **149**(2005) 235–273.

[10] V. Novák: *Perception-based logical deduction*, in: B. Reusch (Ed.), *Computational Intelligence, Theory and Applications*, Springer, Berlin, 2005, pp. 237–250.

[11] V. Novák: *Which logic is the real fuzzy logic?*, Fuzzy Sets and Systems **157** (2006) 635–641.

[12] V. Novák: *A comprehensive theory of trichotomous evaluative linguistic expressions*, Fuzzy Sets and Systems **159** (2008) 2939–2969.

[13] V. Novák: *A formal theory of intermediate quantifiers*, Fuzzy Sets and Systems **159** (2008) 1229–1246.

[14] V. Novák: *EQ-algebra-based fuzzy type theory and its extensions*, Logic Journal of the IGPL **19** (2011) 512–542.

[15] V. Novák, A. Dvořák: *Formalization of commonsense reasoning in fuzzy logic in broader sense*, Journal of Applied and Computational Mathematics **10** (2011) 106–121.

[16] V. Novák, S. Lehmke: *Logical structure of fuzzy IF-THEN rules*, Fuzzy Sets and Systems **157** (2006) 2003–2029.

[17] V. Novák, I. Perfilieva: *On the semantics of perception-based fuzzy logic deduction*, International Journal of Intelligent Systems **19** (2004) 1007–1031.

[18] P. Peterson: *Intermediate Quantifiers. Logic, linguistics, and Aristotelian semantics*, Ashgate, Aldershot, 2000.

[19] P. Vopěnka: *Mathematics in the Alternative Set Theory*, Leipzig, Teubner 1979.