

Perspectives in Model Theory of Fuzzy Logic

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Abstract

Model theory is the branch of mathematical logic that studies the construction and classification of structures. Construction means building structures or families of structures, which have some feature that interest us. Classifying a class of structures means grouping the structures into subclasses in a useful way, and then proving that every structure in the collection does belong in just one of the subclasses.

During the last decades the roadmap of classical model theory has been centered in the development of stability theory (based on Morley's theorem on uncountably categorical theories and Shelah's classification program), which has developed a calculus of independence and rank based on syntactical conditions satisfied by theories. The field of applied model theory has repeatedly merged with the more pure stability theory, the result of this synthesis is called geometric model theory. An example of a theorem from geometric model theory is Hrushovski's proof of the Mordell-Lang conjecture for function fields. The ambition of geometric model theory is to provide a geography of mathematics by embarking on a detailed study of definable sets in various mathematical structures.

Which are the objectives of model theory of fuzzy logic in the next years? Which class of mathematical structures are we interested in? My contribution to the debate about the future of mathematical fuzzy logic will be presenting a brief overview of the state of the art in model theory of fuzzy logic and to raise some questions to be discussed. My personal view is that further research to understand some fundamental concepts such as isomorphism or elementary equivalence is needed; and also that basic methods such as diagrams or type analysis, must be systematically developed to have a solid foundation of the discipline. However, more attention should be devoted to work in finite model theory, the model theory of finite fuzzy structures; and also to understand better our relationship with other disciplines.

As researchers of a cutting-edge area, we need to write a roadmap for mathematical fuzzy logic, identifying a list of important open problems to solve in the years to come; some of these problems being of theoretical nature, others strongly influenced by our interdisciplinary research in some fields such as artificial intelligence or cognitive science.