What is an inconsistent truth table?

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Joint work with G Badia (Otago) and P Girard (Auckland)
Introduction: Non-classical logic, top to bottom

Elements of a (paraconsistent) metatheory

Semantics

Soundness, completeness, and non-triviality

Conclusion
Q: Can standard reasoning about logic be carried out without any appeal to classical logic?
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This is evidence for a more general claim:

Metatheory determines object theory.

When we write down the orthodox clauses for a logic, whatever logic we presuppose in the background will be the object-level logic that obtains.
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When talking about a logic, must we be working in a classical metatheory?
How far can a logician who professes to hold that [paraconsistency] is the correct criterion of a valid argument, but who freely accepts and offers standard mathematical proofs, in particular for theorems about [paraconsistent] logic itself, be regarded as sincere or serious in objecting to classical logic? [Burgess]
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Okay ... then what is the plan for once everyone is converted to the One True (paraconsistent) Logic?
Armchair pop-psychology claim:
Classical-fallback is simply pragmatic.
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No one really knows what e.g. a fully paraconsistently constructed truth table looks like.
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And so the main reason for this paper is pragmatic, too—just to show the answer.
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Here, a paraconsistent set theory naturally generates a paraconsistent semantics.
Logic (Propositional Fragment)

Axioms

\[ 
\vdash \varphi \rightarrow \varphi \\
\vdash (\varphi \rightarrow \psi) \land (\psi \rightarrow \chi) \rightarrow (\varphi \rightarrow \chi)
\]

\[ 
\vdash \varphi \lor \neg \varphi \\
\vdash \neg \neg \varphi \rightarrow \varphi \\
\vdash (\varphi \rightarrow \neg \psi) \rightarrow (\psi \rightarrow \neg \varphi) \\
\vdash \varphi \land \psi \rightarrow \varphi \\
\vdash \varphi \land \psi \rightarrow \psi \land \varphi \\
\vdash \varphi \land \psi \leftrightarrow \neg (\neg \varphi \land \neg \psi) \\
\vdash \varphi \land (\psi \lor \chi) \leftrightarrow (\varphi \land \psi) \lor (\varphi \land \chi)
\]

\[ 
\vdash (\varphi \rightarrow \psi) \Rightarrow (\varphi \Rightarrow \psi) \\
\vdash \neg (\varphi \Rightarrow \psi) \Rightarrow \neg (\varphi \rightarrow \psi) \\
\vdash (\varphi \Rightarrow \psi) \land (\chi \Rightarrow \psi) \Rightarrow (\varphi \lor \chi \Rightarrow \psi)
\]

\[ 
\vdash x = y \Rightarrow (\varphi(x) \rightarrow \varphi(y))
\]
Rules

\[ \varphi, \varphi \Rightarrow \psi \vdash \psi \]
\[ \varphi, \neg \psi \vdash \neg (\varphi \Rightarrow \psi) \]

\[ \Gamma, \varphi \vdash \psi \]
\[ \Gamma, \varphi \Rightarrow \psi \]

\[ \Gamma, \varphi, \chi \vdash \psi \]
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\[ \Gamma, \varphi \vdash \psi \]
\[ \Gamma, \varphi, \chi \vdash \psi \]

\[ \therefore \Gamma, \varphi \land \chi \vdash \psi \]

\[ \Gamma \vdash \psi \]
\[ \Delta \vdash \varphi \]
\[ \therefore \Gamma, \Delta \vdash \varphi \land \psi \]

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Axiom (Ext)
∀z((z ∈ x ⇔ z ∈ y) ⇔ x = y)

Axiom (Abs)
\(x ∈ \{z : \varphi\} ⇔ \varphi x\)
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\[ \forall z ((z \in x \iff z \in y) \iff x = y) \]

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Special case: \[ \langle x, y \rangle \in \{ z : \varphi \} \iff \varphi \langle x, y \rangle \]
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\[ x \in \{z : \varphi\} \leftrightarrow \varphi x \]
Special case: \( \langle x, y \rangle \in \{z : \varphi\} \leftrightarrow \varphi \langle x, y \rangle \)

Axiom (Choice)
A unique object can be picked out from any non-empty set.

Axiom (Induction)
Proofs by induction work for any recursively defined structure.
Two relations are added: syntactic validity $\vdash$ and semantic consequence $\models$. 
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**Definition**

*With $\Gamma$ a set of premises,*

$$\Gamma \vdash \varphi$$

*iff $\varphi$ follows from some subset of $\Gamma$ by valid rules.*
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If this sounds (comfortingly? suspiciously?) familiar, this is prelude for what is to come.
True vs true only: two values or three?

Tarski’s theorem:
An exclusive and exhaustive partitioning of all the propositions into all-and-only the truths, versus all-and-only the non-truths, is impossible.
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*incomplete strategy*
accept ‘only the truths’, leave some out

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Choose: untruth-avoidance or truth-seeking.
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If the original Tarski problem was insoluble, this new, three-tiered approach will be no less intractable.
The three-valued approach rather encourages a common criticism—that dialetheists have lost some important expressive power, the ability to demarcate the truths (t valued) from the true contradictions (b valued).
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“Surely this distinction is available—there it is in your semantics!—but the object language cannot express it.”

Indeed ... if not for the original problem: no one can in fact make this demarcation.

A dialetheic paraconsistentist should lead the discussion away from pre-Tarskian ideation, and use a formalism that does not invite or suggest such criticism.
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Relations can approximate the otherwise-desirable functional three valued semantics;

To reiterate, this is not really a *decision* on our part, but rather a *requirement* for any logic that can express its own metatheory.
There are two truth values, t and f, which are duals,

\[ \overline{t} = f \]

\[ \overline{\overline{t}} = t \]

They are also exclusive, on pain of absurdity:

\[ t = f \Rightarrow \varphi \]

for any \( \varphi \).
Relational Truth Conditions

A truth-value assignment on PROP is any relation

\[ R^0 \subseteq \text{PROP} \times \{t, f\} \]

such that \( x \in \text{PROP} \iff \exists y (\langle x, y \rangle \in R^0) \), and

\[ \langle p, t \rangle \in R^0 \iff \langle p, f \rangle \notin R^0 \]
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By the law of excluded middle, \( R^0 \) is not empty:

either \( \langle p, f \rangle \in R^0 \), or else \( \langle p, f \rangle \notin R^0 \), in which case \( \langle p, t \rangle \in R^0 \).
Definition of a model

Extend $R^0$ to $R \subseteq \text{FMLA} \times \{t, f\}$:

- $\neg \varphi RT \iff \varphi RF$
- $\neg \varphi RF \iff \varphi RT$

- $(\varphi \land \psi) RT \iff \varphi RT \land \psi RT$
- $(\varphi \land \psi) RF \iff \varphi RF \lor \psi RF$

- $(\varphi \lor \psi) RT \iff \varphi RT \lor \psi RT$
- $(\varphi \lor \psi) RF \iff \varphi RF \land \psi RF$
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Extend $R^0$ to $R \subseteq \text{FMLA} \times \{t, f\}$:

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(\varphi \lor \psi) R_t \iff \varphi R_t \lor \psi R_t \\
(\varphi \lor \psi) R_f \iff \varphi R_f \land \psi R_f
\]

Satisfies $\varphi R_t \iff \neg (\varphi R_f)$ and $\varphi R_f \iff \neg (\varphi R_t)$
$R$ is called a *model* for extensional propositional logic.
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$R$ satisfies formula $\varphi$, or $Sat(R, \varphi)$, iff $\langle \varphi, t \rangle \in R$. 

Example: If both $x \varphi$, $t \gamma$, $x \varphi$, $f \gamma \in R$, then $Sat(R, \varphi)$ and $Sat(R, \varphi)$ simultaneously, i.e. $\varphi$ is both satisfied and not in the model $R$. This will be the situation with any contradiction.
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Definition

A sentence $\psi$ is a valid consequence of $\varphi_0, \ldots, \varphi_n$,

$$\varphi_0, \ldots, \varphi_n \models \psi$$

iff $\varphi_0 R_t \land \ldots \land \varphi_n R_t \Rightarrow \psi R_t$ for all models $R$. 
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This is as usual.
Theorem

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Let \( R^0 \subseteq PROP \times \{t, f\} \) be an assignment on propositional variables. This means that

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One exists: let $R^0 = \{\langle p, t \rangle, \langle p, f \rangle \}$. 
Extend $R^0$ with the lift $R$ (which exists by comprehension):

$$
R = \{ \langle p, t \rangle : p R^0 t \} \cup \{ \langle p, f \rangle : p R^0 f \} \\
\cup \{ \langle \neg \varphi, t \rangle : \langle \varphi, f \rangle \in R \} \\
\cup \{ \langle \neg \varphi, f \rangle : \langle \varphi, t \rangle \in R \} \\
\cup \{ \langle \varphi \land \psi, t \rangle : \langle \varphi, t \rangle \in R \land \langle \psi, t \rangle \in R \} \\
\cup \{ \langle \varphi \land \psi, f \rangle : \langle \varphi, f \rangle \in R \lor \langle \psi, f \rangle \in R \} \\
\cup \{ \langle \varphi \lor \psi, t \rangle : \langle \varphi, t \rangle \in R \lor \langle \psi, t \rangle \in R \} \\
\cup \{ \langle \varphi \lor \psi, f \rangle : \langle \varphi, f \rangle \in R \land \langle \psi, f \rangle \in R \}
$$
Proof that R is a model

Show by induction that for any formula

\((\varphi, t) \in R \iff (\varphi, f) \notin R\)

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For the base case, for any proposition \(p\) and \(x \in \{t, f\}\), by definition

\(pRx \iff pR^0x\) and \(-pRx \iff -(pR^0x)\).

Induction: assume \((\star)\) as the inductive hypothesis.

To avoid contraction, we don’t use the very same hypothesis for each inductive case. They are rather hypothesis schemata, each instance used once.
So what does an inconsistent truth table look like?

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<thead>
<tr>
<th></th>
<th>( \neg )</th>
<th>( \land )</th>
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- No such assumption on the page.
- It is simply presupposing classicality to do so.
- Diagrams must be used with great care in mathematics!
The tables are read as, ‘if t is among the values of ϕ, then f is among the values of ¬ϕ’.

Or more concisely, ‘if ϕ is true, then ¬ϕ is false’.
The tables are read as, ‘if $t$ is among the values of $\varphi$, then $f$ is among the values of $\neg \varphi$’.

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The tables are read as, ‘if $t$ is *among* the values of $\varphi$, then $f$ is *among* the values of $\neg\varphi$’.

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The copula—the ‘is’ of predication—is not univocal in general, and it is not here.
Soundness

Theorem
\[ \vdash \varphi \Rightarrow \models \varphi. \]
Also, there are \( \varphi \) such that \( \vdash \varphi \) and \( \not\models \varphi \).
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Corollary

For some \( \varphi \), it is the case that \( \vdash \varphi \Rightarrow \models \varphi \) and \( \neg(\vdash \varphi \Rightarrow \models \varphi) \).

Corollary

\( (\models \varphi \Rightarrow \bot) \Rightarrow (\vdash \varphi \Rightarrow \bot) \).
Completeness

Notation: given $R$, 

$$\Theta^\varphi_R = \{ p_i : \langle p_i, t \rangle \in R \} \cup \{ \neg p_i : \langle p_i, f \rangle \in R \}$$

$\Theta$ contains as many copies of $p_i$ as there are in $\varphi$.

Lemma

For any model $R$ and formula $\varphi$, 

1. $\langle \varphi, t \rangle \in R \implies \Theta^\varphi_R \models \varphi$
2. $\langle \varphi, f \rangle \in R \implies \Theta^\varphi_R \models \neg \varphi$

Proof: If $\langle \neg \psi, t \rangle \in R$, then $\langle \psi, f \rangle \in R$, so $\Theta \models \neg \psi$. If $\langle \neg \psi, f \rangle \in R$, then $\langle \neg \neg \psi, f \rangle \in R$, so $\Theta \models \neg \neg \psi$, so $\Theta \models \psi$. Etc. □

Theorem

$\vdash \varphi \implies \models \varphi$
Non-triviality

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**Theorem**  Naive set theory is not trivial.

**Proof.**
Either naive set theory is trivial or not. If not, we are done. If trivial, then, since this very proof is in naive set theory, it follows that the system is not trivial—since, after all, anything follows. □
Non-triviality

In general, a theory is *non-trivial* iff there is at least one sentence that is not part of the theory.

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Indeed, in a paraconsistent system, one *can* prove consistency and non-triviality.
In classical mathematics, consistency (and so non-triviality) is not provable.

Using a paraconsistent metatheory constitutes no expressive loss. Rather it makes clearer and more explicit the hard facts.

Indeed, in a paraconsistent system, one can prove consistency and non-triviality.

This is the closest one can get to a guarantee that the proof methods themselves are reliable, by methods that are equally reliable.
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If you bring to the uninterpreted propositional connectives a presupposition of classical logic, then the connectives will be classical.
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This hardly shows that metatheory ‘must’ be conducted in classical language!
In logic, you get out what you put in

Logic does not tell us what is true. It tells us what is true, *given* some other truths.

If you bring to the uninterpreted propositional connectives a presupposition of classical logic, then the connectives will be classical.

This hardly shows that metatheory ‘must’ be conducted in classical language!

A paraconsistent substructural approach can at least match the classical textbook presentation of semantics, and may eventually be uniquely able to carry its own weight.