

The Future of MFL
Prague, June 16-18, 2016

The future of MFL: Pure math or seriously interdisciplinary?

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In fact, this stage has now been reached in a number of branches of fuzzy set theory.

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- ▶ “...rethink the research directions of MFL”
– This requires thinking “outside the box”!

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Leaving the “gilded cage” of pure math does not come easy!

A suggestion:

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- (1) modeling reasoning with (vague) natural language
- (2) justifications, consequences, and limits of truth functionality
- (3) fuzzy logics as logics of costs
- (4) efficient reasoning with graded truth

Modeling natural language

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Different applications call for **different modeling principles**
How to move from **ad hoc modeling** to **'first principles'**?

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- voting semantics (t.f. **explained** by levels of skepticism)
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- approximation semantics (vis-a-vis probabilistic reasoning)
- game semantics (various forms)

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Such semantics might provide an **interface** of MFL to applications.

But – again – this calls for **leaving the gilded cage** of pure math!

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Note the interdisciplinarity!

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Probably most challenging:

- ▶ concrete proof tasks, arising from applications
- ▶ implementing efficient provers
- ▶ comparisons, bench marks

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Taking this challenge seriously should be on the agenda of MFL!