On the role of Mathematical Fuzzy Logic in Knowledge Representation

Francesc Esteva, Lluís Godo,
IIIA - CSIC, Barcelona, Spain

The Future of Mathematical Fuzzy Logic, Prague, June 16-17, 2016

Graded formalisms from an AI perspective

- One heavily entrenched tradition in AI, especially in KRR, is to rely on Boolean logic. However, many epistemic notions in common-sense reasoning are perceived as gradual rather than all-or-nothing

- Many logical formalisms in AI designed to allow an explicit representation of quantitative or qualitative weights associated with classical or modal logical formulas

Graded formalisms from an AI perspective

- One heavily entrenched tradition in AI, especially in KRR, is to rely on Boolean logic. However, many epistemic notions in common-sense reasoning are perceived as gradual rather than all-or-nothing
- Many logical formalisms in AI designed to allow an explicit representation of quantitative or qualitative weights associated with classical or modal logical formulas
- Large variety of intended meanings of weights or degrees:
  - truth degrees
  - belief degrees,
  - preference degrees,
  - trust degrees,
  - similarity degrees
  - ...

Graded formalisms from an AI perspective

A number of weighted/graded formalisms have been developed for KRR:

- fuzzy logics, including:
  - fuzzy logic programs under various semantics
  - fuzzy description logics
- probabilistic and possibilistic uncertainty logics,
- preference logics,
- weighted computational argumentation systems,
- logics handling inconsistency degrees
- etc.

Graded formalisms from an AI perspective

A number of weighted/graded formalisms have been developed for KRR:

- fuzzy logics, including:
  - fuzzy logic programs under various semantics
  - fuzzy description logics
- probabilistic and possibilistic uncertainty logics,
- preference logics,
- weighted computational argumentation systems,
- logics handling inconsistency degrees
- etc.

But not all graded logics are fuzzy logics !

A brief landscape on

basic graded notions in KR models

- **Uncertainty**
- **Preference**
- **Similarity**

with special emphasis on

- **Truth**

A brief landscape on

Uncertainty

Uncertainty modeling is about the representation of an agent's beliefs.

A brief landscape on

Uncertainty

Uncertainty modeling is about the representation of an agent's beliefs.

Several kinds of reasons for the presence of uncertainty:

- Random variation of a class of repeatable events
- Lack of information
- Inconsistent pieces of information

A brief landscape on

Uncertainty

Uncertainty modeling is about the representation of an agent's beliefs.

Several kinds of reasons for the presence of uncertainty:

- Random variation of a class of repeatable events
- Lack of information
- Inconsistent pieces of information

Two main traditions in AI for representing uncertainty:

- The non-graded Boolean tradition of epistemic modal logics and exception-tolerant non-monotonic logics.
- The graded tradition typically relying on degrees of probability (and more generally on measures of uncertainty)

  Choice of a proper scale: ordinal, finite, integer-valued, real-valued

A brief landscape on

Uncertainty

Uncertainty modeling is about the representation of an agent's beliefs.

Several kinds of reasons for the presence of uncertainty:

- Random variation of a class of repeatable events
- Lack of information
- Inconsistent pieces of information

Two main traditions in AI for representing uncertainty:

- The non-graded Boolean tradition of epistemic modal logics and exception-tolerant non-monotonic logics.
- The graded tradition typically relying on degrees of probability (and more generally on measures of uncertainty)

  Choice of a proper scale: ordinal, finite, integer-valued, real-valued

Uncertainty is a non-compositional, higher-order notion wrt truth:
"I believe p" (regardless whether p is true)

A brief landscape on

Preferences

The tradition in preference modeling has been to use either order relations (total or partial) or numerical utility functions

However, AI has focused on compact logical (à la von Wright, $\varphi P \psi$) or graphical representations (CP nets) of preferences on multi-dimensional domains with Boolean attributes, leading to orderings in the set of interpretations (= options, solutions, configurations)

A brief landscape on

Preferences

The tradition in preference modeling has been to use either order relations (total or partial) or numerical utility functions

However, AI has focused on compact logical (à la von Wright, $\varphi P \psi$) or graphical representations (CP nets) of preferences on multi-dimensional domains with Boolean attributes, leading to orderings in the set of interpretations ($=$ options, solutions, configurations)

An alternative is using weights attached to Boolean formulas with possible different semantics: priorities, rewards, utilitarian desires, etc.
$\Rightarrow$ different orderings on interpretations

A brief landscape on

Preferences

The tradition in preference modeling has been to use either order relations (total or partial) or numerical utility functions

However, AI has focused on compact logical (à la von Wright, $\varphi P \psi$) or graphical representations (CP nets) of preferences on multi-dimensional domains with Boolean attributes, leading to orderings in the set of interpretations (= options, solutions, configurations)

An alternative is using weights attached to Boolean formulas with possible different semantics: priorities, rewards, utilitarian desires, etc. $\Rightarrow$ different orderings on interpretations

When both uncertainty and preferences are present (like in DU), two kinds of weights:

- Weights expressing preferences of options over other ones.
- Weights expressing the likelihood of events or importance of groups of criteria.

Reasoning = optimization of a given criterion mixing uncertainty and utility.

A brief landscape on

Similarity

Similarity in reasoning is useful for:

- differentiating inside a set of objects that are found to be similar
  $\Rightarrow$ granulation of the universe (rough sets, fuzzy partitions)
- taking advantage of the closeness of objects with respect to others
  $\Rightarrow$ extrapolation or interpolation

A brief landscape on

Similarity

Similarity in reasoning is useful for:

- differentiating inside a set of objects that are found to be similar
  $\Rightarrow$ granulation of the universe (rough sets, fuzzy partitions)
- taking advantage of the closeness of objects with respect to others
  $\Rightarrow$ extrapolation or interpolation

Similarity is often a *graded* notion, especially when it is related to the idea of distance. It may refer:

- to a physical space, as in spatial reasoning (graded extensions of RCC, modal logic approach for upper/lower approximations), or
- to an abstract space used for describing similar situations, as in CBR (closeness between interpretations, similarity-based approximate reasoning)

A brief landscape on

Similarity

Similarity in reasoning is useful for:

- differentiating inside a set of objects that are found to be similar $\Rightarrow$ granulation of the universe (rough sets, fuzzy partitions)
- taking advantage of the closeness of objects with respect to others $\Rightarrow$ extrapolation or interpolation

Similarity is often a *graded* notion, especially when it is related to the idea of distance. It may refer:

- to a physical space, as in spatial reasoning (graded extensions of RCC, modal logic approach for upper/lower approximations), or
- to an abstract space used for describing similar situations, as in CBR (closeness between interpretations, similarity-based approximate reasoning)

Also qualitative approaches: using comparative relations (e.g. Sheremet's CSL binary modal operators), Gärdenfors' conceptual spaces for interpolation/extrapolation, or analogical proportions (Prade et al.)

A brief landscape on

Graded Truth

- Although the truth of a proposition is usually viewed as Boolean, it is a matter of convention (De Finetti)

- In some contexts the truth of a proposition (understood as its conformity with a *precise description of the state of affairs*) is a matter of degree: *gradual* properties like in “The room is large”

A brief landscape on

Graded Truth

- Although the truth of a proposition is usually viewed as Boolean, it is a matter of convention (De Finetti)
- In some contexts the truth of a proposition (understood as its conformity with a *precise description of the state of affairs*) is a matter of degree: *gradual* properties like in "The room is large"
- Presence of intermediate degrees of truth: truth-functionality leads to many-valued / fuzzy logics.
- But most popular ones in AI are 3-valued Kleene, 4-valued Belnap or 5-valued equilibrium logics that deal with epistemic notions (e.g. ignorance, contradiction or negation as failure), at odds with truth-functionality ...

A closer look: MFL and KR

In principle MFL appears as an ideal, well-founded and deeply developed formalism to model reasoning with imprecise / gradual / incomplete information, that is pervasive in any AI real application domain.

A closer look: MFL and KR

In principle MFL appears as an ideal, well-founded and deeply developed formalism to model reasoning with imprecise / gradual / incomplete information, that is pervasive in any AI real application domain.

If this is so, it should have been much more used as a key tool in knowledge representation.

A closer look: MFL and KR

In principle MFL appears as an ideal, well-founded and deeply developed formalism to model reasoning with imprecise / gradual / incomplete information, that is pervasive in any AI real application domain.

If this is so, it should have been much more used as a key tool in knowledge representation.

But many unresolved issues from an applicative point of view, e.g.:

- How to choose among the many available systems?
- Does truth-functionality always make sense?
- Why so few papers using (mathematical) fuzzy logics for KR in main AI conferences?

A closer look: MFL and KR

Are there actually so few papers on (mathematical) fuzzy logic in AI conferences?

A closer look: MFL and KR

Are there actually so few papers on (mathematical) fuzzy logic in AI conferences?

IJCAI conferences are the top AI conferences

- 2007, 2009: no paper at all
- 2011: 3 out of 400
  Reasoning about Fuzzy Belief and Common Belief: With Emphasis on Incomparable Beliefs
  Description Logics over Lattices with Multi-Valued Ontologies
  Finite-Valued Lukasiewicz Modal Logic is PSPACE-Complete
- 2013: 2 out of 413
  Positive Subsumption in Fuzzy EL with General t-Norms
  Syntactic Labelled Tableaux for Łukasiewicz Fuzzy ALC
- 2015: 1 out of 572 papers
  The Complexity of Subsumption in Fuzzy EL

A closer look: MFL and KR

Why is so?

A closer look: MFL and KR

Why is so?

- Many-valued logics have been seriously criticized at the philosophical level because of the confusion between truth-values and degrees of belief,
  —a confusion going back to pioneers including Łukasiewicz with the idea of possible as a third truth-value—

A closer look: MFL and KR

Why is so?

- Many-valued logics have been seriously criticized at the philosophical level because of the confusion between truth-values and degrees of belief,
  —a confusion going back to pioneers including Łukasiewicz with the idea of possible as a third truth-value—
- Actually, due to this and the numerical flavor of fuzzy logic, there is a long tradition of mutual distrust between AI practitioners and fuzzy logic

A closer look: MFL and KR

Why is so?

- Many-valued logics have been seriously criticized at the philosophical level because of the confusion between truth-values and degrees of belief,
  —a confusion going back to pioneers including Łukasiewicz with the idea of possible as a third truth-value—
- Actually, due to this and the numerical flavor of fuzzy logic, there is a long tradition of mutual distrust between AI practitioners and fuzzy logic
- Not a clear what is the added-value by moving from a two-valued to a graded, fuzzy logic model:
  gain in expressivity, but usually an increase of complexity and a lack of efficient reasoning tools (like SAT, CSP, ASP, etc.)
  – fuzzy description logics!

A closer look: MFL and KR

How to improve the situation and bridge gaps?

A closer look: MFL and KR

How to improve the situation and bridge gaps?

Difficult question, I can only think of some naïve proposals:

A closer look: MFL and KR

How to improve the situation and bridge gaps?

Difficult question, I can only think of some naïve proposals:

- Find AI-related applications that naturally require the use of logic formalisms with graded truth (Rosta's famous list?)

A closer look: MFL and KR

How to improve the situation and bridge gaps?

Difficult question, I can only think of some naïve proposals:

- Find AI-related applications that naturally require the use of logic formalisms with graded truth (Rosta's famous list?)
- Use MFL to encode (graded modalities that can account for) various uncertainty, preference, similarity theories; these graded modalities may be applied to Boolean formulas, yielding two-tiered logic formalisms (Petr-Carles' second layer?)

A closer look: MFL and KR

How to improve the situation and bridge gaps?

Difficult question, I can only think of some naïve proposals:

- Find AI-related applications that naturally require the use of logic formalisms with graded truth (Rosta's famous list?)
- Use MFL to encode (graded modalities that can account for) various uncertainty, preference, similarity theories; these graded modalities may be applied to Boolean formulas, yielding two-tiered logic formalisms (Petr-Carles' second layer?)
- Show how (defeasible) reasoning about knowledge, uncertainty, preferences, etc., can also be defined on top of fuzzy/gradual propositions by augmenting fuzzy logic with epistemic modalities. (MFL as a unifying formalism: Petr-Carles' third layer?)

Concluding remarks

- Personal perception: MFL does not play the role it would deserve in KRR / AI / Computer Science

Concluding remarks

- Personal perception: MFL does not play the role it would deserve in KRR / AI / Computer Science

- "We live in a golden age of computer science and computing research is transforming society as we know it. Do we view ourselves as a scientific discipline divorced from these changes, or should our theory play a major role? This is a discussion and debate the community should continue to have. "

Concluding remarks

- Personal perception: MFL does not play the role it would deserve in KRR / AI / Computer Science

- "We live in a golden age of computer science and computing research is transforming society as we know it. Do we view ourselves as a scientific discipline divorced from these changes, or should our theory play a major role? This is a discussion and debate the community should continue to have. "

  Probably this could be a conclusion of the workshop on the future of MFL

Concluding remarks

- Personal perception: MFL does not play the role it would deserve in KRR / AI / Computer Science

- “We live in a golden age of computer science and computing research is transforming society as we know it. Do we view ourselves as a scientific discipline divorced from these changes, or should our theory play a major role? This is a discussion and debate the community should continue to have. ”

  Probably this could be a conclusion of the workshop on the future of MFL

  http://blog.computationalcomplexity.org/2016/06/karp-v-wigderson-20-years-later.html