Fuzzy Logic and Psychology

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The Future of Mathematical Fuzzy Logic

Prague · 16–18 June 2016

(a)

Zadeh logic

$$
\begin{aligned}
[\neg\alpha] &= & & 1 - [\alpha] \\
[\alpha \wedge \beta] &= & & \mathsf{min}\{[\alpha], [\beta]\} \\
[\alpha \vee \beta] &= & & \mathsf{max}\{[\alpha], [\beta]\} \\
[\alpha \rightarrow \beta] &= [\neg\alpha \vee \beta] &=& [\neg(\alpha \wedge \neg\beta)]
\end{aligned}
$$

Fine I

Suppose that a certain blob is on the border of pink and red.

- P: the blob is pink
- R: the blob is red

P and R are neither clearly true nor clearly false.
Fine:

- $P \vee R$ is clearly true
- $P \wedge R$ is clearly false.

Osherson & Smith I

Any object at all should be a member of

things that are and are not apples to degree 0

and of

things that are or are not apples to degree 1.

Kamp

$\alpha \wedge \neg\alpha$ should be true to degree 0, even when $[\alpha] = 0.5$

$\alpha \wedge \alpha$ should be true to a degree > 0, when $[\alpha] = 0.5$

Assuming $[\neg\alpha] = 1 - [\alpha]$, no truth function for $\wedge$ can predict this.

Fine II

With P and R as in Fine I, Fine claims:

- $P \wedge P$ is equivalent to P
  hence neither clearly true nor clearly false

but $P \wedge R$ is clearly false.

Osherson & Smith II

(a)

The degree to which (a) is an apple is less than the degree to which (a) is a striped apple.

Other Intuitions

> [Fine] claims that 'red' and 'pink', even though vague and admitting of borderline cases of applicability, are nevertheless logically connected so that to say of some color shade that it is both red and pink is obviously to say something false. I must confess being completely insensitive to that intuition of a penumbral connection.
>
> Machina 1976

Support for the present position is provided by the observation that, as Osherson & Smith themselves remark in a footnote (p.45), people do in fact frequently use locutions such as (of tomatoes) ‘They are both fruit and not fruit’.

contrary to traditional binary taxonomy but in accordance with the present view, a concept such as ape that is not an ape is indeed not an empty one.

Jones 1982

An aspect of the theory of fuzzy sets which Osherson & Smith find objectionable is that, in the theory, the union of A and its complement, A', is not, in general, the whole universe of discourse. This relates, of course, to the long-standing controversy regarding the validity of the principle of the excluded middle

The principle of the excluded middle is not accepted as a valid axiom in the theory of fuzzy sets because it does not apply to situations in which one deals with classes which do not have sharply defined boundaries.

Zadeh 1982

the failure of contradiction and excluded-middle laws is typical of fuzzy logic as emphasized by many authors. This is natural with gradual properties like ‘tall’.

Dubois and Prade 1994

Łukasiewicz logic

$$
\begin{aligned}
x \wedge y &= \max(0, x + y - 1) \\
x \rightarrow y &= \begin{cases} 1 & \text{if } x \leq y \\ 1 - x + y & \text{if } x > y \end{cases} \\
\neg x &= 1 - x
\end{aligned}
$$

Fine I

In Łukasiewicz logic, when $[P] = [R] = 0.5$:

- $[P \vee R] = 1$
- $[P \wedge R] = 0$

(where $\wedge$ is the Łukasiewicz t-norm and $\vee$ is its dual)

Osherson & Smith I

In Łukasiewicz logic:

- $[\alpha \wedge \neg\alpha] = 0$
- $[\alpha \vee \neg\alpha] = 1$

(whatever the degree of truth of α)

Gödel logic

$$\begin{aligned}
x \wedge y &= \min(x, y) \\
x \to y &= \begin{cases} 1 & \text{if } x \leq y \\ y & \text{if } x > y \end{cases} \\
\neg x &= \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases}
\end{aligned}$$

Product logic

$$
\begin{aligned}
x \wedge y &= x \cdot y \\
x \rightarrow y &= \begin{cases} 1 & \text{if } x \leq y \\ y/x & \text{if } x > y \end{cases} \\
\neg x &= \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
$$

Kamp

In Gödel logic, when $[\alpha] = 0.5$:

- $[\alpha \wedge \neg\alpha] = 0$
- $[\alpha \wedge \alpha] = 0.5$

In product logic, when $[\alpha] = 0.5$:

- $[\alpha \wedge \neg\alpha] = 0$
- $[\alpha \wedge \alpha] = 0.25$

Fine II

In Łukasiewicz logic, when $[P] = [R] = 0.5$:

- P and P is 0.5 true where and is read as weak conjunction
- P and R is 0 true where and is read as strong conjunction

Osherson & Smith II

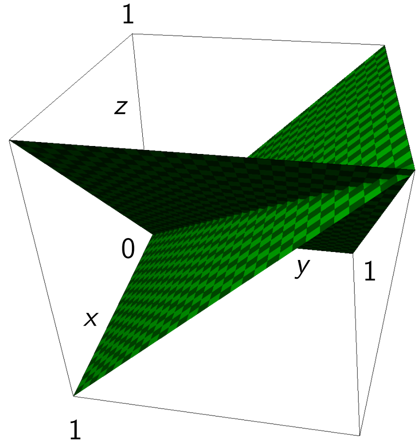


Figure: min and max

Empirical Work

1. Single concepts
2. Combinations of concepts

Oden 1977

presented pairs of simple predications and had subjects judge:

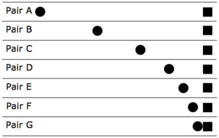
1. the average degree of truthfulness of the two statements
2. the degree to which one statement or the other was true
3. the degree to which both statements were true.

Concerning 3:

> *of the 32 subjects, 23 were fit best by the multiplying rule, whereas only 8 were fit best by the averaging rule and only 1 was fit best by the minimum rule.*

Ripley 2011

projected a slide with seven circle/square pairs on it and asked subjects to indicate their level of agreement (on a scale from 1 to 7, with 1 labelled Disagree and 7 labelled Agree) with a given compound sentence as applied to each pair.

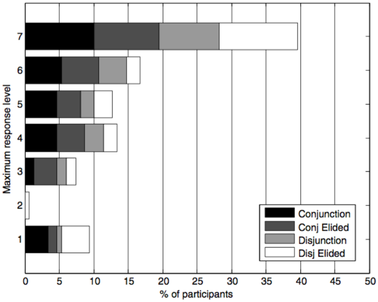


Conjunction, Non-elided: The circle is near the square and it isn't near the square.
Conjunction, Elided: The circle both is and isn't near the square.
Disjunction, Non-elided: The circle neither is near the square nor isn't near the square.
Disjunction, Elided: The circle neither is nor isn't near the square.

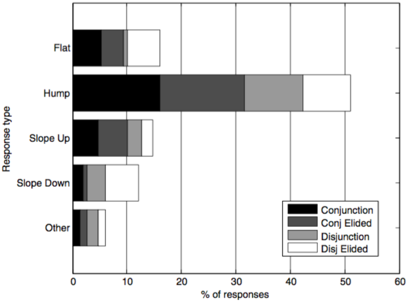
Ripley noted four distinct patterns of response:

- Flat (24/149 participants):
  A flat response gives the same number for every question.
- Slope up (22/149 participants):
  A slope up response is not a flat response, and it never goes down from question to question.
- Slope down (18/149 participants):
  A slope down response is not a flat response, and it never goes up from question to question.
- Hump (76/149 participants):
  A hump response is not a flat response or a slope response, and it has a peak somewhere between the first and last question; before the peak, responses never go down from question to question, and after the peak, responses never go up from question to question.

9 participants fitted none of these four patterns.



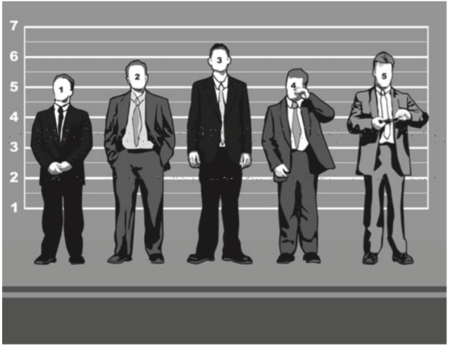
7
6
5
4
3
2
1
Maximum response level
0
5
10
15
20
25
30
35
40
45
50
% of participants
Conjunction
Conj Elided
Disjunction
Disj Elided



Flat
Hump
Slope Up
Slope Down
Other
Response type
0
10
20
30
40
50
60
% of responses
Conjunction
Conj Elided
Disjunction
Disj Elided

Alxatib and Pelletier 2011

presented subjects with an image



and asked them to respond—True, False or Can't Tell—to sentences about each man.

Out of the five sentences:

- #1 is tall and not tall
- #2 is tall and not tall
- #3 is tall and not tall
- #4 is tall and not tall
- #5 is tall and not tall

the second received the greatest number of True responses: 44.7% of subjects responded True to '#2 is tall and not tall'.

Serchuk, Hargreaves and Zach 2011

asked subjects to imagine that on the spectrum of rich women, Susan is somewhere between women who are clearly rich and women who are clearly non-rich and then asked subjects for their response ('true', 'not true, but also not false', 'partially true and partially false', 'false', 'both true and false' or 'true or false, but I don't know which') to sentences including:

1. Susan is rich and Susan is not rich.
2. Susan is rich and it is not the case that Susan is rich.

19% responded 'true' to 1 and 7% responded 'true' to 2.

They note: These results differ greatly from those found by Alxatib and Pelletier.