Energy Complexity Model for Convolutional Neural Networks

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Efficient Processing of Deep Neural Networks (DNNs)

- DNNs are widely used for many artificial intelligence (AI) applications including computer vision, speech recognition, natural language processing, robotics etc.

- DNNs achieve state-of-the-art accuracy on many AI tasks at the cost of high computational complexity (tens of millions of operations for a single inference)

- energy efficiency of DNN implementations in low-power hardware operated on batteries (e.g. cellphones, smartwatches, smart glasses) becomes crucial

$\longrightarrow$ reducing the energy cost of DNNs:

1. approximate computing methods (e.g. low floating-point precision, approximate multipliers) in error-tolerant applications such as image classification

2. hardware design: energy-efficient implementations of DNNs on various hardware platforms including GPUs, FPGAs, in-memory computing architectures

Energy Consumption of DNNs

- the power consumption of a specific DNN hardware implementation can be measured or calculated/estimated (using physical laws)
- a plethora of methods that minimize the energy consumption of a given DNN on various hardware architectures
(Sze,Chen,Yang,Emer:Efficient Processing of Deep Neural Networks,2020)
- automated by software tools, for example, the `Timeloop` program maps a convolutional layer specified by its parameters onto a given hardware architecture (e.g. Simba, Eyeriss) that is optimal in terms of power consumption estimated by `Accelergy` tool which reports the energy statistics
- it has been empirically observed that the energy for DNN inference is mainly consumed by
  1. data movement inside a memory hierarchy (approx. 70%) corresponding to the data energy E_{data}
  2. multiply-and-accumulate (MAC) operations (approx. 30%): $S \leftarrow S + wx$ on floats S, w, x, corresponding to the computation energy E_{comp}

$$\longrightarrow \quad E = E_{\text{data}} + E_{\text{comp}}$$

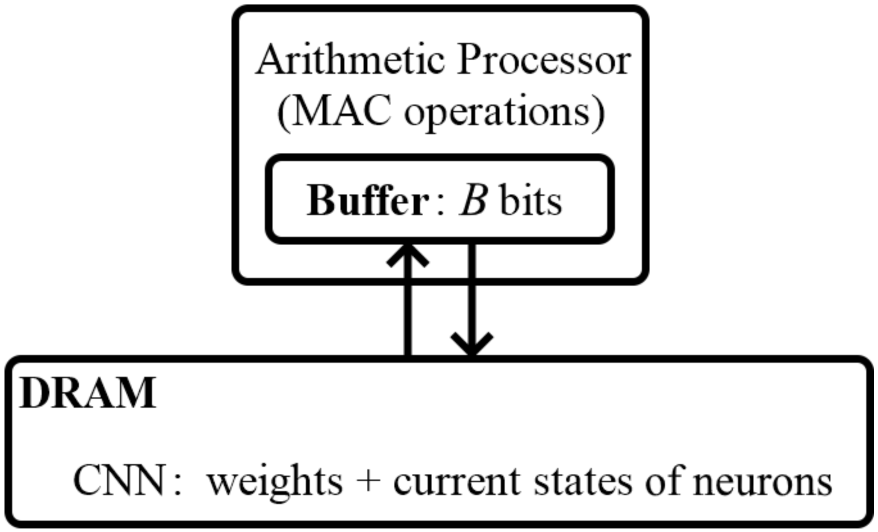
Motivations for Energy Complexity Model of DNNs

- the evaluation of real power consumption for individual DNN implementations varies for different hardware architectures depending on their specific parameters, which prevents from machine-independent exploration of energy complexity
- a formal computational model for defining a robust energy measure for DNNs, quantified asymptotically using Big O notation
  (by analogy to computation time and memory space defined by Turing machines)
- lower bounds on energy complexity can establish principal limits of DNNs

$\longrightarrow$ A Simplified Hardware-Independent Model of Energy Complexity for DNNs:

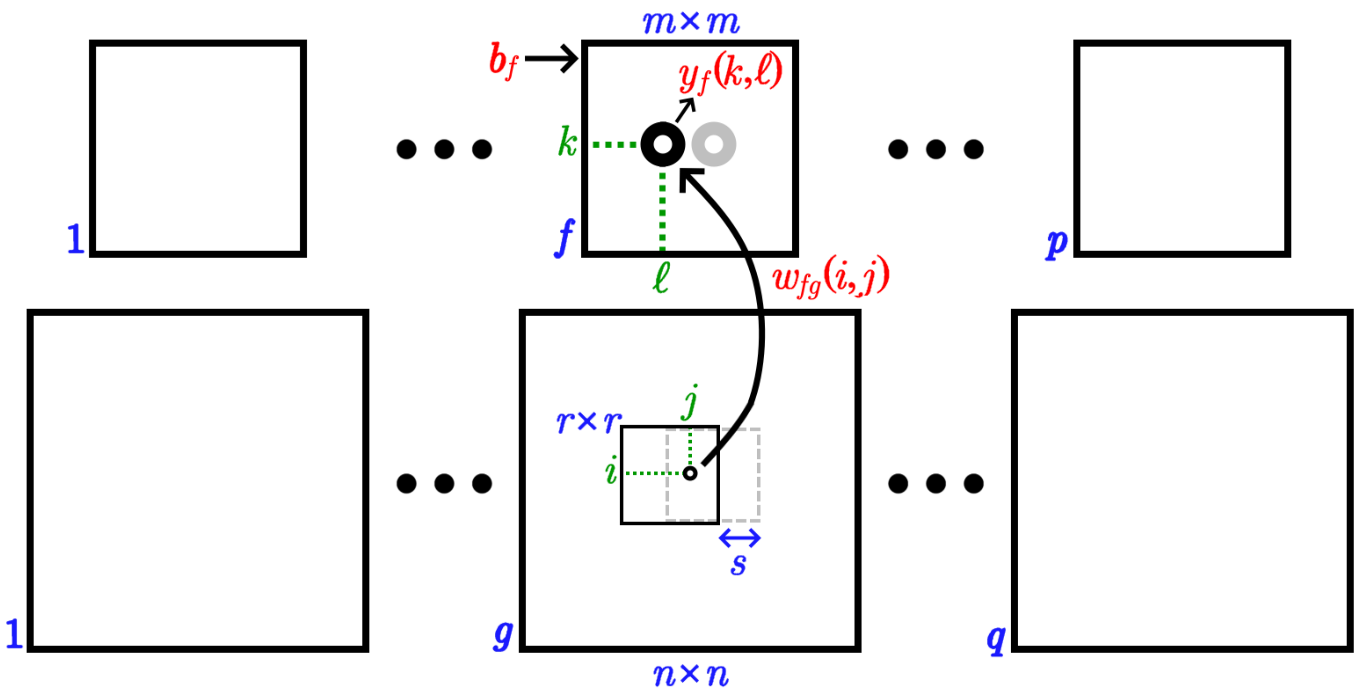
- abstracts from hardware implementation details, ignoring specific aspects and parameters of real-world machine
- preserves the asymptotic energy of DNN inference
- is defined (for simplicity) for a separate layer of a convolutional neural network (CNN), avoiding global energy optimization across multiple CNN layers

Energy Complexity Model for CNNs



- only two memory levels called DRAM (large, slow, and cheap memory) and Buffer of limited capacity B bits (small, fast, and expensive memory)
- CNN weights and states are stored in DRAM
- arithmetic operations are performed over numerical data stored in Buffer
- the dataflow controls the transfer of data between DRAM and Buffer
- the main idea: the three arguments stored in DRAM, input x, weight w, and accumulated output S of each MAC operation $S \leftarrow S + wx$ performed for evaluating a given convolutional layer, must occur in Buffer simultaneously

A Convolutional Layer



p feature maps $f \in \{1, \ldots, p\}$
of size $m \times m$ neurons
(depth $= p$, height=width $= m$)

receptive fields
of size $r \times r$ neurons
with stride s

q input feature maps
$g \in \{1, \ldots, q\}$
of size $n \times n$ neurons
$m \approx \frac{n}{s}$

$y_f(k, \ell)$ is the state of neuron $(k, \ell) \in \{1, \ldots, m\}^2$ in feature map $f \in \{1, \ldots, p\}$

$$y_f(k,\ell) = \mathrm{ReLU}\left(b_f + \sum_{g=1}^{q} \sum_{i=1}^{r} \sum_{j=1}^{r} w_{fg}(i,j) \cdot y_g\big((k-1)s + i\,,\,(\ell-1)s + j\big) \right)$$

where $\mathrm{ReLU}(x) = \max(0, x)$, b_f is the bias of f, and $w_{fg}(i,j)$ is the filter weight of neuron $(i,j) \in \{1, \ldots, r\}^2$ in a receptive field of f over the input feature map $g \in \{1, \ldots, q\}$

$\longrightarrow$ the number of MAC operations (#MACs) is $p\,m^2 \cdot q\,r^2 \;\approx\; p\,q\,n^2\,\frac{r^2}{s^2}$

The Energy Complexity Measure for a Convolutional Layer

$$E = E_{\text{data}} + E_{\text{comp}}$$

for a given dataflow used to evaluate a convolutional layer:

E_{data} is # DRAM accesses $\times$ # bits b in floating-point numbers

$E_{\text{comp}} = C_b \cdot p\,q\,m^2 r^2 \approx p\,q\,n^2\,\frac{r^2}{s^2}$ is proportional to # MACs (on data in Buffer)

where C_b is a non-uniform constant related to a b-bit floating-point MAC circuit

A Simple Lower Bound on the Data Energy

$E_{\text{data}} \;\geq\; b \cdot$ # MACs divided by $\left(\frac{B-1}{2}\right)^2 =$ the maximum number of new triplets (input, output, weight) (i.e. the MAC arguments) that can meet in Buffer of capacity B bits after reading one number into Buffer (i.e. one DRAM access)

$$\longrightarrow \quad E_{\text{data}} = \Omega\left(p\,q\,n^2\,\frac{r^2}{s^2}\right) \quad \text{for constant Buffer capacity } B$$

A Partition of the Input Feature Map

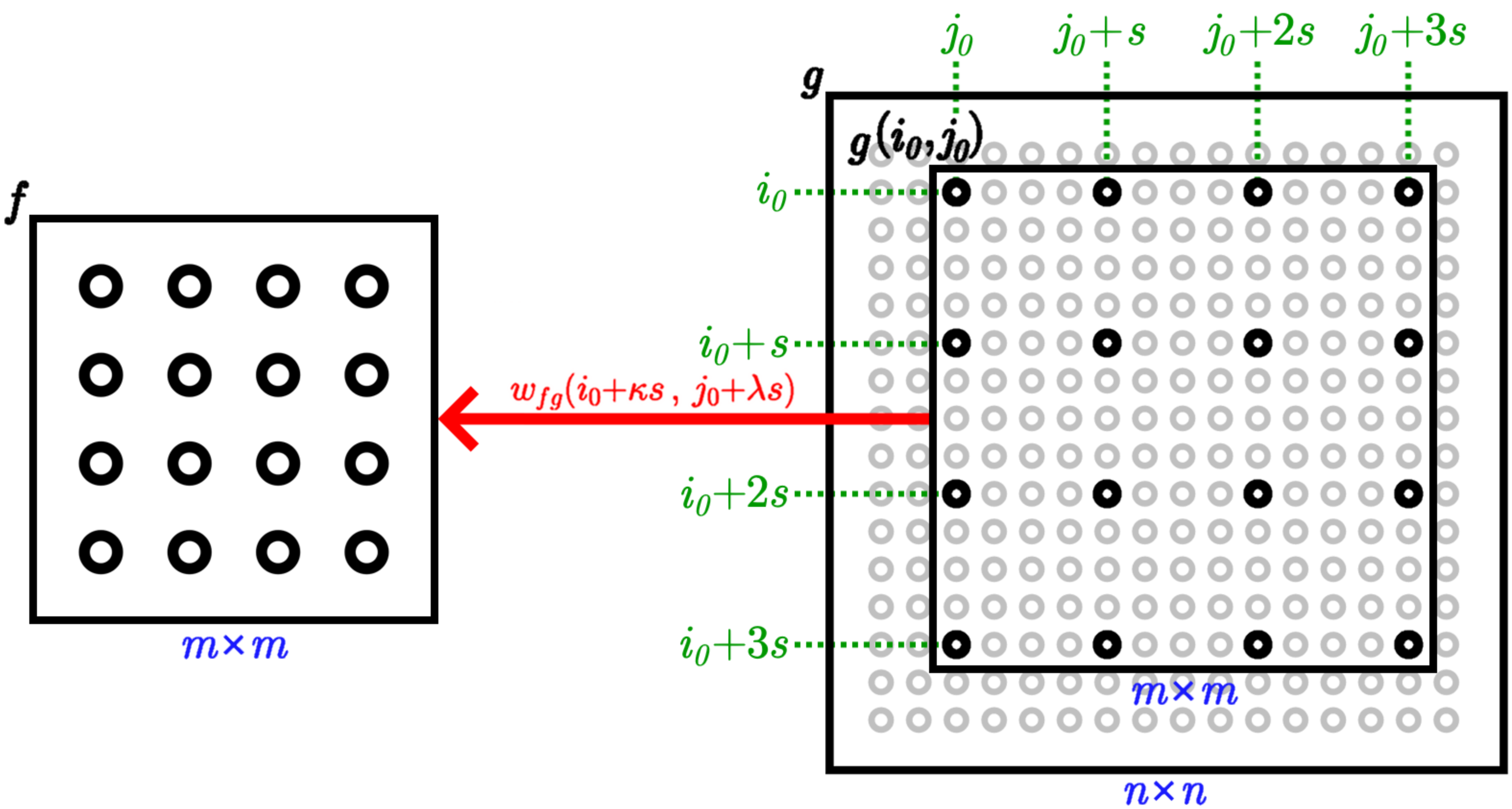
a partition of input feature map $g \in \{1, \ldots, q\}$ of $n \times n$ neurons,

$$g = \bigcup_{i_0, j_0 \in \{1,\ldots,s\}} g(i_0\,, j_0) \quad \text{into } s^2 \text{ grid submaps}$$

$$g(i_0, j_0) = \Big\{ \big((k-1)s + i_0\,,\ (\ell-1)s + j_0 \big) \mid k, \ell \in \{1, \ldots, m\} \Big\} \text{ of } m \times m \text{ neurons}$$

that share the same weights $w_{fg}(i_0 + \kappa s\,,\ j_0 + \lambda s)$ for every (admissible) integer κ, λ

$$\text{in} \quad y_f(k, \ell) = \mathrm{ReLU}\left(b_f + \sum_{g=1}^{q} \sum_{i=1}^{r} \sum_{j=1}^{r} w_{fg}(i, j) \cdot y_g\big((k-1)s + i\,,\ (\ell-1)s + j\big) \right)$$



A Dataflow with Write-Once Outputs (similarly for read-once inputs)

each output is completely evaluated at once in `Buffer` before it is written to `DRAM` while each weights is read into `Buffer` only once:

for all feature maps $f \in \{1, \ldots, p\}$ **do**
  read bias b_f into `Buffer`;
  for all $k, \ell \in \{1, \ldots, m\}$ **do** $S_f(k, l) \leftarrow b_f$ **enddo**;   {initialization of $m \times m$ weighted sums}
  for all input feature maps $g \in \{1, \ldots, q\}$ **do**
    for all $i_0, j_0 \in \{1, \ldots, s\}$ **do**   {for all grid submaps $g(i_0, j_0)$ from the partition of g}
      for all $(k, \ell) \in g(i_0, j_0)$ **do read** $y_g(k, \ell)$ into `Buffer` **enddo**;   {reading $m \times m$ submap inputs}
      for all admissible integer κ, λ **do**   {for all the weights shared by submap $g(i_0, j_0)$}
        $i \leftarrow i_0 + \kappa s$; $j \leftarrow j_0 + \lambda s$; $\{1 \leq i, j \leq r\}$
        read a single weight $w_{fg}(i, j)$ into `Buffer`;
        for all $k, \ell \in \{1, \ldots, m\}$ **do**   {all MACs with the weight $w_{fg}(i, j)$}
          $S_f(k, l) \leftarrow S_f(k, l) + w_{fg}(i, j) \cdot y_g\big((k-1)s + i\,,\,(\ell - 1)s + j\big)$
        enddo
      enddo   {next shared weight}
    enddo   {next submap}
  enddo;   {next input feature map}
  for all $k, \ell \in \{1, \ldots, m\}$ **do write** $y_f(k, l) = \mathrm{ReLU}\big(S_f(k, l)\big)$ to `DRAM` **enddo** {writing $m \times m$ outputs}
enddo   {next feature map}

The Capacity of `Buffer`

the used `Buffer` memory: $B = b \cdot (2m^2 + 1)$

- m^2 accumulated outputs of feature map f
- m^2 inputs from grid submap $g(i_0, j_0)$
- 1 shared weight

$\longrightarrow$ a realistic assumption on the `Buffer` capacity:

$$B \geq b \cdot (2m^2 + 1)$$

e.g. `Buffer` capacities in kilobytes required for convolutional layers in AlexNet:

AlexNet layer	1	2	3	4	5
m	55	27	13	13	13
$2m^2 + 1$	6051	1459	339	339	339
$b = 8$ bits	5.91 kB	1.42 kB	0.33 kB	0.33 kB	0.33 kB
$b = 16$ bits	11.82 kB	2.85 kB	0.66 kB	0.66 kB	0.66 kB
$b = 32$ bits	23.64 kB	5.7 kB	1.32 kB	1.32 kB	1.32 kB

An Upper Bound on Data Energy E_{data}

the data energy E_{data} in terms of #DRAM accesses for inputs, outputs, and weights:

$$E_{\text{data}} = E_{\text{weights}} + E_{\text{outputs}} + E_{\text{inputs}} \qquad \text{where}$$

$$E_{\text{inputs}} = b \cdot p\, q\, n^2 \qquad E_{\text{outputs}} = b \cdot p\, m^2 \approx b \cdot p\, \frac{n^2}{s^2} \qquad E_{\text{weights}} = b \cdot p(q\, r^2 + 1)$$

$\longrightarrow$ an upper bound:

$$E_{\text{data}} \;\leq\; b \cdot p\,(q\, n^2 + m^2 + q\, r^2 + 1) = O\left(p\left(qn^2 + \frac{n^2}{s^2} + qr^2\right)\right)$$

the asymptotic theoretical energy complexity in terms of individual convolutional layer parameters (others are constant):

$E_{\text{data}} = O(p)$ where p is the number of feature maps (i.e. depth)

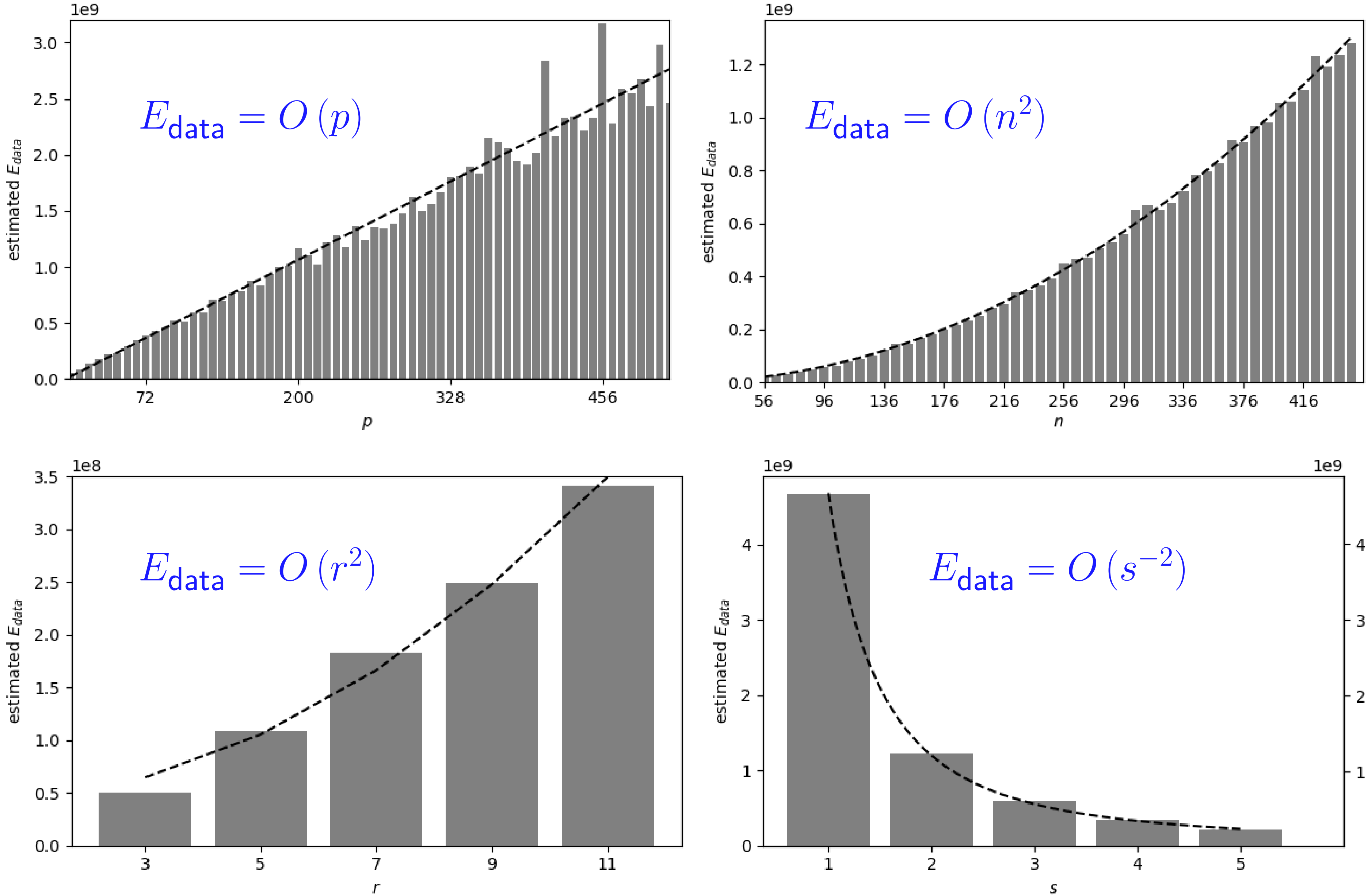
$E_{\text{data}} = O(n^2)$ where n is the the size of input feature maps (i.e. height=width)

$E_{\text{data}} = O(r^2)$ where r is the size of receptive fields

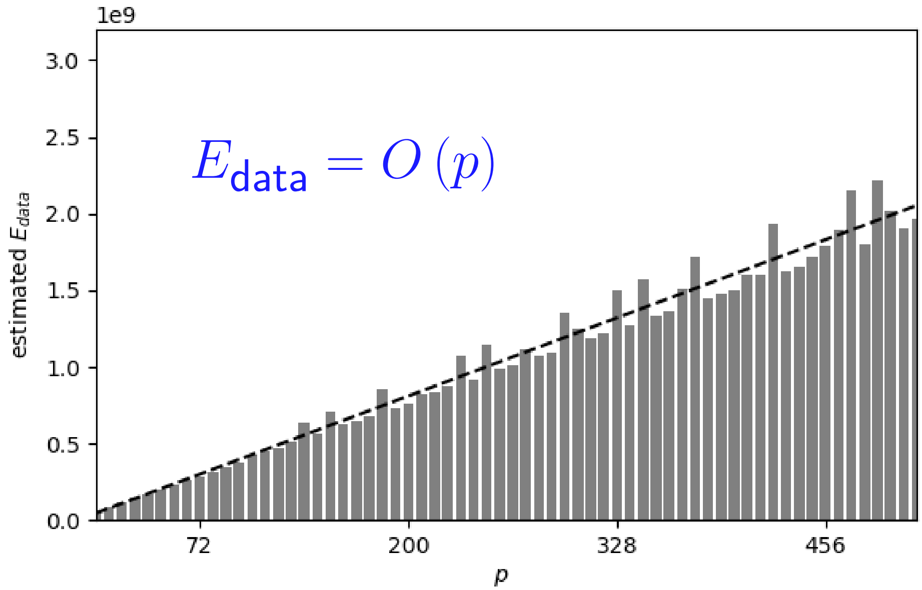
$E_{\text{data}} = O(s^{-2})$ where s is the stride

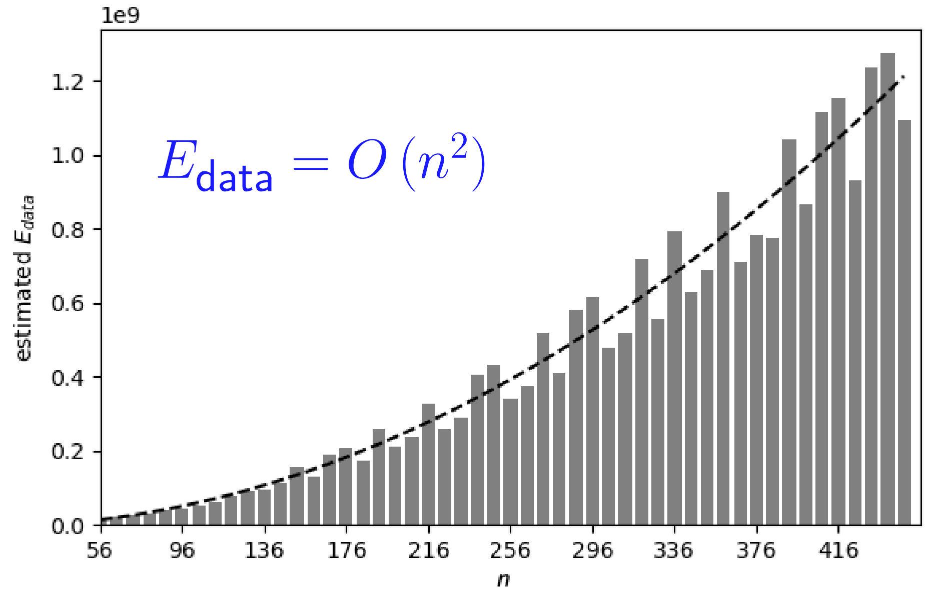
fits very well (by linearity/quadraticity statistical tests) the real power consumptions estimated by the `Timeloop/Accelergy` software platform that maps a convolutional layer of given parameters onto the Simba and Eyeriss hardware architectures:

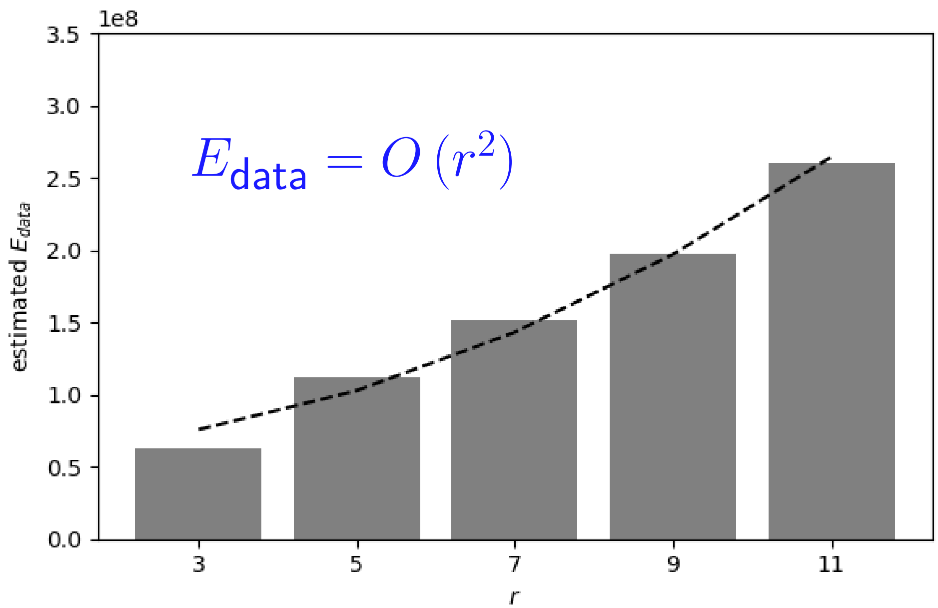
Experimental Validation of Energy Complexity Model for Simba

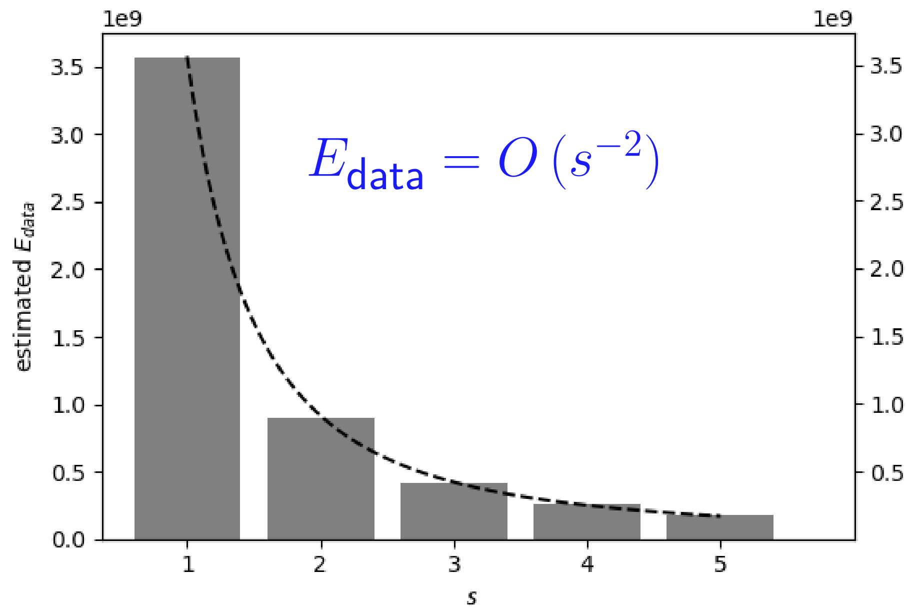


Experimental Validation of Energy Complexity Model for Eyeriss









A Summary

- we have introduced a machine-independent model of energy complexity for CNNs
- in this model, we have proposed a dataflow with write-once outputs (or read-once inputs) and read-once weights for evaluating convolutional layers
- this provides an upper bound on the theoretical energy complexity of CNNs which fits asymptotically very well the power consumption estimates of their various hardware implementations
- we have shown a simple lower bound on energy of convolutional layers which establishes the principal limit on energy efficiency of CNNs

Open Problems

- an experimental validation of the energy complexity model for combined parameters of convolutional layer **?**
- the matching lower bound on energy of convolutional layers for `Buffer` of non-constant size **?**